Chapter 10. Isosceles Triangles

Exercise 10(A)

Solution 1:

In ΔABC,

$\angle BAC + \angle ACB + \angle ABC = 180^0$

$48^0 + \angle ACB + \angle ABC = 180^0$

But $\angle ACB = \angle ABC$ [AB = AC]

$2\angle ABC = 180^0 - 48^0$

$2\angle ABC = 132^0$

$\angle ABC = 66^0 = \angle ACB$ ......(i)

$\angle ACB = 66^0$

$\angle ACD + \angle DCB = 66^0$

$18^0 + \angle DCB = 66^0$

$\angle DCB = 48^0$ ....(ii)

Now, In ΔDCB,

$\angle DBC = 66^0$ [From (i), Since $\angle ABC = \angle DBC$]

$\angle DCB = 48^0$ [From (ii)]

$\angle BDC = 180^0 - 48^0 - 66^0$

$\angle BDC = 66^0$

Since $\angle BDC = \angle DBC$

Therefore, BC = CD

Equal angles have equal sides opposite to them.

Solution 2:

Given: $\angle ACE = 130^0$; AD = BD = CD

Proof:

(i)

$\angle ACD + \angle ACE = 180^\circ$ [DCE is a st. line]

$\Rightarrow \angle ACD = 180^\circ - 130^\circ$

$\Rightarrow \angle ACD = 50^\circ$

Now, CD = AD

$\Rightarrow \angle ACD = \angle DAC = 50^\circ$.....(i)

[Since angles opposite to equal sides are equal]

In ΔADC,

$\angle ACD = \angle DAC = 50^\circ$

$\angle ACD + \angle DAC + \angle ADC = 180^\circ$

$50^\circ + 50^\circ + \angle ADC = 180^\circ$

$\angle ADC = 180^\circ - 100^\circ$

$\angle ADC = 80^\circ$

(ii)

$\angle ADC = \angle ABD + \angle DAB$ [Exterior angle is equal to sum of opp. interior angles]

But AD = BD

$\therefore \angle DAB = \angle ABD$

$\Rightarrow 80^\circ = \angle ABD + \angle ABD$

$\Rightarrow 2\angle BD = 80^\circ$

$\Rightarrow \angle ABD = 40^\circ = \angle DAB$..........(ii)

(iii)

$\angle BAC = \angle DAB + \angle DAC$

substituting the values from (i) and (ii)

$\angle BAC = 40^\circ + 50^\circ$

$\Rightarrow \angle BAC = 90^\circ$

Solution 3:

$\angle FAB = 128°$ [Given]

$\angle BAC + \angle FAB = 180°$ [FAC is a st. line]

$\Rightarrow \angle BAC = 180° - 128°$

$\Rightarrow \angle BAC = 52°$

In ΔABC,

$\angle A = 52°$

$\angle B = \angle C$ [Given AB = AC and angles opposite to equal sides are equal]

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle A + \angle B + \angle B = 180°$

$\Rightarrow 52° + 2\angle B = 180°$

$\Rightarrow 2\angle B = 128°$

$\Rightarrow \angle B = 64° = \angle C$ ....(i)

$\angle B = \angle ADE$ [Given DE || BC]

(i)

Now,

$\angle ADE + \angle CDE + \angle B = 180°$ [ADB is a st. line]

$\Rightarrow 64° + \angle CDE + 64° = 180°$

$\Rightarrow \angle CDE = 180° - 128°$

$\Rightarrow \angle CDE = 52°$

(ii)

Given DE||BC and DC is the transversal.

$\Rightarrow \angle CDE = \angle DCB = 52°$.......(ii)

Also, $\angle ECB = 64°$.........[From (i)]

But,

$\angle ECB = \angle DCE + \angle DCB$

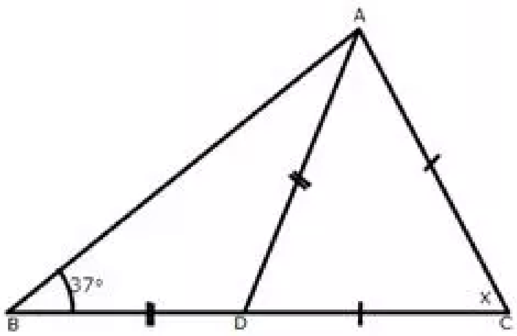
$\Rightarrow 64° = \angle DCE + 52°$

$\Rightarrow \angle DCE = 64° - 52°$

$\Rightarrow \angle DCE = 12°$

Solution 4:

(i) Let the triangle be ABC and the altitude be AD.



In ΔABD,

$\angle DBA = \angle DAB = 37°$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opp. interior angles]

$\therefore \angle CDA = 37° + 37°$

$\Rightarrow \angle CDA = 74°$

Now in ΔADC,

$\angle CDA = \angle CAD = 74°$ [Given CD = AC and angles opposite to equal sides are equal]

Now,

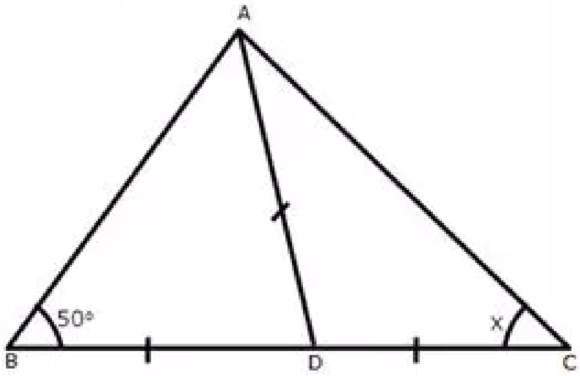
$\angle CAD + \angle CDA + \angle ACD = 180°$

$\Rightarrow 74° + 74° + x = 180°$

$\Rightarrow x = 180° - 148°$

$\Rightarrow x = 32°$

(ii) Let triangle be ABC and altitude be AD.



In ΔABD,

$\angle DBA = \angle DAB = 50°$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opp. interior angles]

$\therefore \angle CDA = 50° + 50°$

$\Rightarrow \angle CDA = 100°$

In ΔADC,

$\angle DAC = \angle DCA = x$ [Given AD = DC and angles opposite to equal sides are equal]

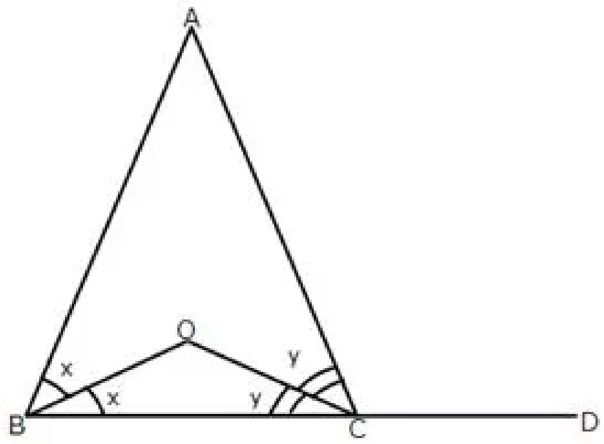
$\therefore \angle DAC + \angle DCA + \angle ADC = 180°$

$\Rightarrow x + x + 100° = 180°$

$\Rightarrow 2x = 80°$

$\Rightarrow x = 40°$

Solution 5:



Let $\angle ABO = \angle OBC = x$ and $\angle ACO = \angle OCB = y$

In ΔABC,

$\angle BAC = 180° - 2x - 2y$.........(i)

Since $\angle B = \angle C$ [AB = AC]

$\frac{1}{2}B = \frac{1}{2}C$

$\Rightarrow x = y$

Now,

$\angle ACD = 2x + \angle BAC$ [Exterior angle is equal to sum of opp. interior angles]

$= 2x + 180° - 2x - 2y$ [From (i)]

$\angle ACD = 180° - 2y$..........(ii)

In ΔOBC,

$\angle BOC = 180° - x - y$

$\Rightarrow \angle BOC = 180° - y - y$ [Already proved]

$\Rightarrow \angle BOC = 180° - 2y$....(iii)

From (i) and (ii)

$\angle BOC = \angle ACD$

Solution 6:

Given: $\angle PLN = 110^\circ$

(i) We know that the sum of the measure of all the angles of a quadrilateral is 360°.

In quad. PQNL,

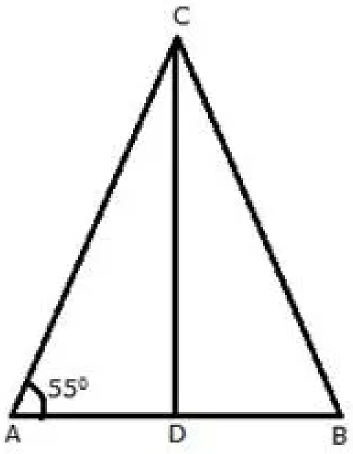
$\angle QPL + \angle PLN + \angle LNQ + \angle NQP = 360^\circ$
$\Rightarrow 90^\circ + 110^\circ + \angle LNQ + 90^\circ = 360^\circ$
$\Rightarrow \angle LNQ = 360^\circ - 290^\circ$
$\Rightarrow \angle LNQ = 70^\circ$
$\Rightarrow \angle LNM = 70^\circ$..............(i)

In ΔLMN,
LM = LN [Given]
$\therefore \angle LNM = \angle LMN$ [angles opp. to equal sides are equal]
$\Rightarrow \angle LMN = 70^\circ$.........(ii) [From (i)]

(ii)

In ΔLMN,
$\angle LMN + \angle LNM + \angle MLN = 180^\circ$
But, $\angle LNM = \angle LMN = 70^\circ$ [From (i) and (ii)]
$\therefore 70^\circ + 70^\circ + \angle MLN = 180^\circ$
$\Rightarrow \angle MLN = 180^\circ - 140^\circ$
$\Rightarrow \angle MLN = 40^\circ$

Solution 7:



In ΔABC,

$AC = BC$ [Given]

$\therefore \angle CAB = \angle CBD$ [angles opp. to equal sides are equal]

$\Rightarrow \angle CBD = 55^\circ$

In ΔABC,

$\angle CBA + \angle CAB + \angle ACB = 180^\circ$

but, $\angle CAB = \angle CBA = 55^\circ$

$\Rightarrow 55^\circ + 55^\circ + \angle ACB = 180^\circ$

$\Rightarrow \angle ACB = 180^\circ - 110^\circ$

$\Rightarrow \angle ACB = 70^\circ$

Now,

In ΔACD and ΔBCD,

$AC = BC$ [Given]

$CD = CD$ [Common]

$AD = BD$ [Given : CD bisects AB]

$\therefore \Delta ACD \cong \Delta BCD$

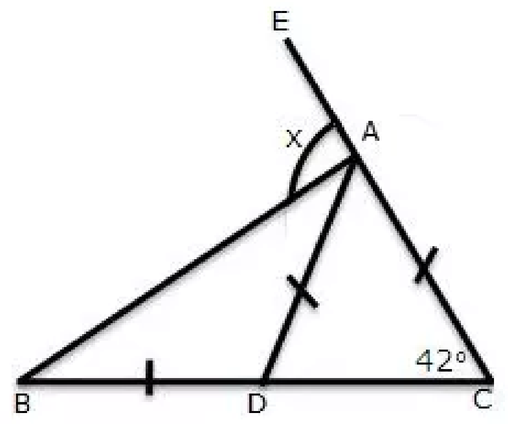
$\Rightarrow \angle DCA = \angle DCB$

$\Rightarrow \angle DCB = \dfrac{\angle ACB}{2} = \dfrac{70^\circ}{2}$

$\Rightarrow \angle DCB = 35^\circ$

Solution 8:

Let us name the figure as following:



In ΔABC,

$AD = AC$ [Given]

$\therefore \angle ADC = \angle ACD$ [angles opp. to equal sides are equal]

$\Rightarrow \angle ADC = 42^\circ$

Now,

$\angle ADC = \angle DAB + \angle DBA$ [Exterior angle is equal to the sum of opp. interior angles]

But,

$\angle DAB = \angle DBA$ [Given : BD = DA]

$\therefore \angle ADC = 2\angle DBA$

$\Rightarrow 2\angle DBA = 42^\circ$

$\Rightarrow \angle DBA = 21^\circ$

For x:

$x = \angle CBA + \angle BCA$ [Exterior angle is equal to the sum of opp. interior angles]

We know that,

$\angle CBA = 21^\circ$

$\angle BCA = 42^\circ$

$\therefore x = 21^\circ + 42^\circ$

$\Rightarrow x = 63^\circ$

Solution 9:

In ΔABD and ΔDBC,

$BD = BD$ [Common]

$\angle BDA = \angle BDC$ [each equal to 90°]

$\angle ABD = \angle DBC$ [BD bisects $\angle ABC$]

$\therefore \Delta ABD \cong \Delta DBC$ [ASA criterion]

Therefore,

AD=DC

$x + 1 = y + 2$

$\Rightarrow x = y + 1 \ldots..(i)$

and AB = BC

$3x + 1 = 5y - 2$

Substituting the value of x from (i)

$3(y + 1) + 1 = 5y - 2$

$\Rightarrow 3y + 3 + 1 = 5y - 2$

$\Rightarrow 3y + 4 = 5y - 2$

$\Rightarrow 2y = 6$

$\Rightarrow y = 3$

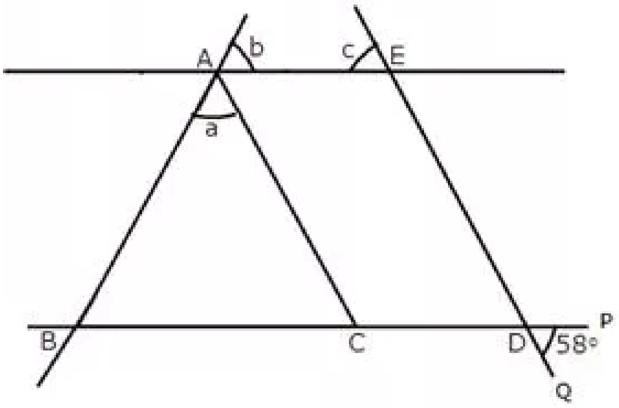
Putting y = 3 in (i)

$x = 3 + 1$

$\therefore x = 4$

Solution 10:

Let P and Q be the points as shown below:



Given: $\angle PDQ = 58°$

$\angle PDQ = \angle EDC = 58°$ [Vertically opp. angles]

$\angle EDC = \angle ACB = 58°$ [Corresponding angles $\because$ AC || ED]

In $\triangle ABC$,

AB = AC [Given]

$\therefore \angle ACB = \angle ABC = 58°$ [angles opp. to equal sides are equal]

Now,

$\angle ACB + \angle ABC + \angle BAC = 180°$

$\Rightarrow 58° + 58° + a = 180°$

$\Rightarrow a = 180° - 116°$

$\Rightarrow a = 64°$

Since AE || BD and AC is the transversal

$\angle ABC = b$ [Corresponding angles]

$\therefore b = 58°$

Also since AE || BD and ED is the transversal

$\angle EDC = c$ [Corresponding angles]

$\therefore c = 58°$

Solution 11:

In ΔACD,

$AC = CD$ [Given]

$\therefore \angle CAD = \angle CDA$

$\angle ACD = 58°$ [Given]

$\angle ACD + \angle CDA + \angle CAD = 180°$

$\Rightarrow 58° + 2\angle CAD = 180°$

$\Rightarrow 2\angle CAD = 122°$

$\Rightarrow \angle CAD = \angle CDA = 61°$..........(i)

Now,

$\angle CDA = \angle DAB + \angle DBA$ [Ext. angle is equal to sum of opp. int. angles]

But,

$\angle DAB = \angle DBA$ [Given : AD = DB]

$\therefore \angle DAB + \angle DAB = \angle CDA$

$\Rightarrow 2\angle DAB = 61°$

$\Rightarrow \angle DAB = 30.5°$..........(ii)

In ΔABC,

$\angle CAB = \angle CAD + \angle DAB$

$\therefore \angle CAB = 61° + 30.5°$

$\Rightarrow \angle AB = 91.5°$

Solution 12:

In ΔACD,

$AC = AD = CD$ [Given]

Hence, ACD is an equilateral triangle

$\therefore \angle ACD = \angle CDA = \angle CAD = 60°$

$\angle CDA = \angle DAB + \angle ABD$ [Ext. angle is equal to sum of opp. int. angles]

But,

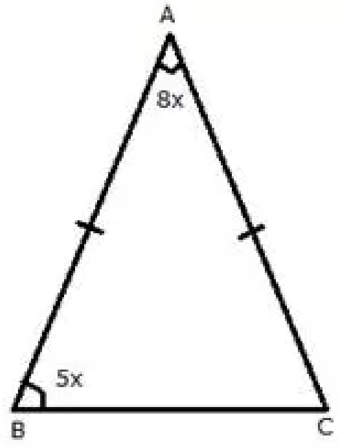
$\angle DAB = \angle ABD$ [Given : AD = DB]

$\therefore \angle ABD + \angle ABD = \angle CDA$

$\Rightarrow 2\angle ABD = 60°$

$\Rightarrow \angle ABD = \angle ABC = 30°$

Solution 13:



Let $\angle A = 8x$ and $\angle B = 5x$

Given: AB = AC

$\Rightarrow \angle B = \angle C = 5x$ [Angles opp. to equal sides are equal]

Now,

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 8x + 5x + 5x = 180°$

$\Rightarrow 18x = 180°$

$\Rightarrow x = 10°$

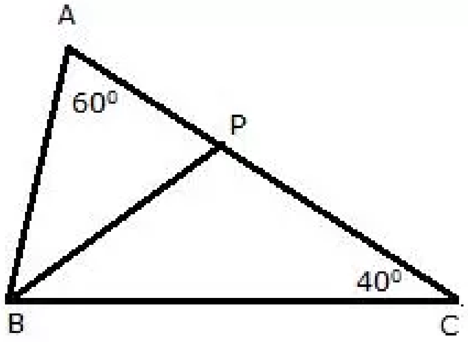
Given that :

$\angle A = 8x$

$\Rightarrow \angle A = 8 \times 10°$

$\Rightarrow \angle A = 80°$

Solution 14:



In ΔABC,

$\angle A = 60°$

$\angle C = 40°$

$\therefore \angle B = 180° - 60° - 40°$

$\Rightarrow \angle B = 80°$

Now,

BP is the bisector of $\angle ABC$

$\therefore \angle PBC = \frac{\angle ABC}{2}$

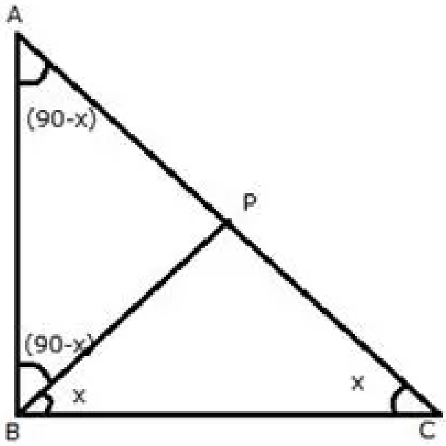
$\Rightarrow \angle PBC = 40°$

In ΔPBC,

$\angle PBC = \angle PCB = 40°$

$\therefore BP = CP$ [Sides opp. to equal angles are equal]

Solution 15:



Let $\angle PBC = \angle PCB = x$

In the right angled triangle ABC,

$\angle ABC = 90^\circ$
$\angle ACB = x$
$\Rightarrow \angle BAC = 180^\circ - (90^\circ + x)$
$\Rightarrow \angle BAC = (90^\circ - x)$.........(i)

and

$\angle ABP = \angle ABC - \angle PBC$
$\Rightarrow \angle ABP = 90^\circ - x$........(ii)

Therefore in the triangle ABP;

$\angle BAP = \angle ABP$

Hence,

PA = PB [sides opp. to equal angles are equal]

Solution 16:

ΔABC is an equilateral triangle

$\Rightarrow$ Side AB = Side AC

$\Rightarrow \angle ABC = \angle ACB$ [If two sides of a triangle are equal, then angles opposite to them are equal]

Similarly, Side AC = Side BC

$\Rightarrow \angle CAB = \angle ABC$ [If two sides of a triangle are equal, then angles opposite to them are equal]

Hence, $\angle ABC = \angle CAB = \angle ACB = y$(say)

As the sum of all the angles of the triangle is 180°

$\angle ABC + \angle CAB + \angle ACB = 180^0$

$\Rightarrow 3y = 180°$

$\Rightarrow y = 60°$

$\angle ABC = \angle CAB = \angle ACB = 60°$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$\Rightarrow \angle CAB + \angle CBA = \angle ACE$

$\Rightarrow 60° + 60^0 = \angle ACE$

$\Rightarrow \angle ACE = 120°$

Now ΔACE is an isosceles triangle with AC = CF

$\Rightarrow \angle EAC = \angle AEC$

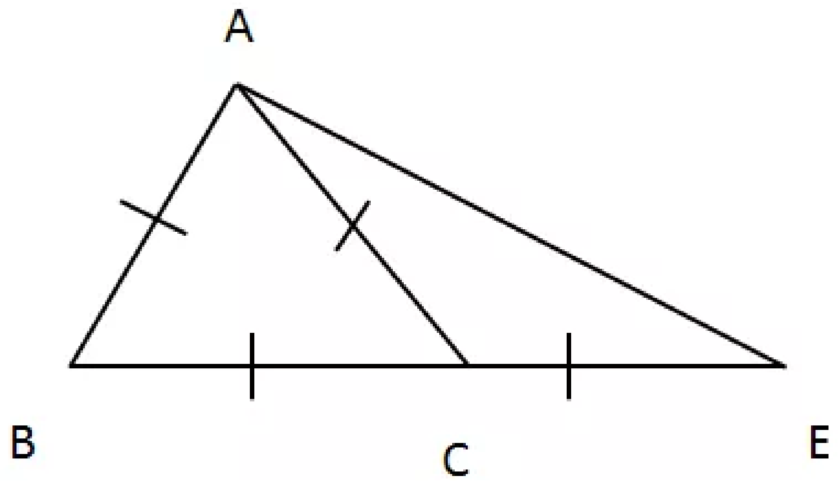
Sum of all the angles of a triangle is 180°

$\angle EAC + \angle AEC + \angle ACE = 180°$

$\Rightarrow 2\angle AEC + 120° = 180°$

$\Rightarrow 2\angle AEC = 180° - 120°$

$\Rightarrow \angle AEC = 30°$



Solution 17:

ΔDBC is an isosceles triangle

As, Side CD = Side DB

$\Rightarrow \angle DBC = \angle DCB$ [If two sides of a triangle are equal, then angles opposite to them are equal]

And $\angle B = \angle DBC = \angle DCB = 28^\circ$

As the sum of all the angles of the triangle is 180°

$\angle DCB + \angle DBC + \angle BCD = 180^0$

$\Rightarrow 28^\circ + 28^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - 56^\circ$

$\Rightarrow \angle BCD = 124^\circ$

Sum of two non-adjacent interior angles of a triangle is equal to the exterior angle.

$\Rightarrow \angle DBC + \angle DCB = \angle DAC$

$\Rightarrow 28^\circ + 28^\circ = 56^\circ$

$\Rightarrow \angle DAC = 56^\circ$

Now ΔACD is an isosceles triangle with AC = DC

$\Rightarrow \angle ADC = \angle DAC = 56^\circ$

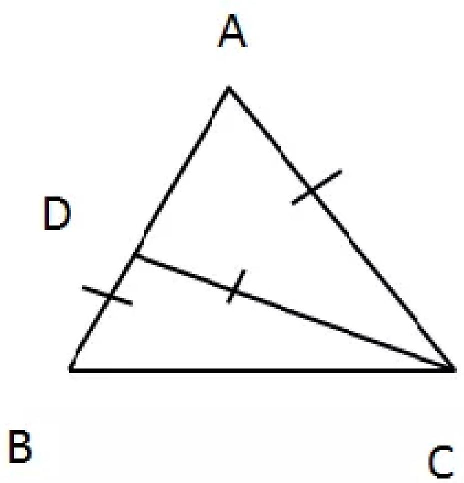
Sum of all the angles of a triangle is 180°

$\angle ADC + \angle DAC + \angle DCA = 180^\circ$

$\Rightarrow 56^\circ + 56^\circ + \angle DCA = 180^\circ$

$\Rightarrow \angle DCA = 180^\circ - 112^\circ$

$\Rightarrow \angle DCA = 64^\circ = \angle ACD$



Solution 18:

We can see that the ΔABC is an isosceles triangle with Side AB = Side AC.

$\Rightarrow \angle ACB = \angle ABC$

As $\angle ACB = 65°$

hence $\angle ABC = 65°$

Sum of all the angles of a triangle is 180°

$\angle ACB + \angle CAB + \angle ABC = 180°$

$65° + 65° + \angle CAB = 180°$

$\angle CAB = 180° - 130°$

$\angle CAB = 50°$

As BD is parallel to CA

Therefore, $\angle CAB = \angle DBA$ since they are alternate angles.

$\angle CAB = \angle DBA = 50°$

We see that ΔADB is an isosceles triangle with Side AD = Side AB.

$\Rightarrow \angle ADB = \angle DBA = 50°$

Sum of all the angles of a trianlge is 180°

$\angle ADB + \angle DAB + \angle DBA = 180°$

$50° + \angle DAB + 50° = 180°$

$\angle DAB = 180^0 - 100° = 80°$

$\angle DAB = 80°$

The angle DAC is sum of angle DAB and CAB.

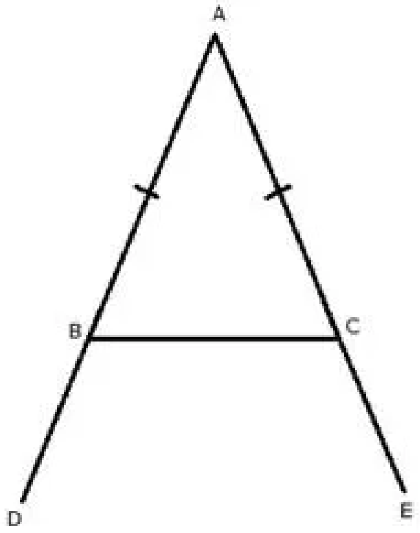
$\angle DAC = \angle CAB + \angle DAB$

$\angle DAC = 50° + 80°$

$\angle DAC = 130°$

Exercise 10(B)

Solution 1:



Const: AB is produced to D and AC is produced to E so that exterior angles $\angle DBC$ and $\angle ECB$ is formed.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$.......(i) [angles opp. to equal sides are equal]

Since angle B and angle C are acute they cannot be right angles or obtuse angles.

$\angle ABC + \angle DBC = 180^\circ$ [ABD is a st. line]

$\Rightarrow \angle DBC = 180^\circ - \angle ABC$

$\Rightarrow \angle DBC = 180^\circ - \angle B$........(ii)

Similarly,

$\angle ACB + \angle ECB = 180^\circ$ [ABD is a st. line]

$\Rightarrow \angle ECB = 180^\circ - \angle ACB$

$\Rightarrow \angle ECB = 180^\circ - \angle C$.........(iii)

$\Rightarrow \angle ECB = 180^\circ - \angle B$.........(iv) [from (i) and (iii)]

$\Rightarrow \angle DBC = \angle ECB$ [from (ii) and (iv)]

Now,

$\angle DBC = 180° - \angle B$

But $\angle B$ = Acute angle

$\therefore \angle DBC = 180°$ – Acute angle = obtuse angle

Similarly,

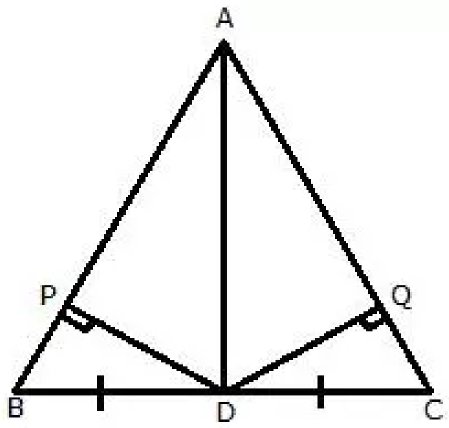
$\angle ECB = 180° - \angle C.$

But $\angle C$ = Acute angle

$\therefore \angle ECB = 180°$ – Acute angle = obtuse angle

Therefore, exterior angles formed are obtuse and equal.

Solution 2:



Const: Join AD.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

(i)

In ΔBPD and ΔCQD,

$\angle BPD = \angle CQD$ [Each = 90°]

$\angle B = \angle C$ [proved]

$BD = DC$ [Given]

$\therefore \Delta BPD \cong \Delta CQD$ [AAS criterion]

$\therefore DP = DQ$ [cpct]

(ii) We have already proved that $\Delta BPD \cong \Delta CQD$

Therefore, $BP = CQ$ [cpct]

Now,

$AB = AC$ [Given]

$\Rightarrow AB - BP = AC - CQ$

$\Rightarrow AP = AQ$

(iii)

In ΔAPD and ΔAQD,

$DP = DQ$ [proved]

$AD = AD$ [common]

$AP = AQ$ [Proved]

$\therefore \Delta APD \cong \Delta AQD$ [SSS]

$\Rightarrow \angle PAD = \angle QAD$ [cpct]

Hence, AD bisects angle A.

Solution 3:

(i)

In ΔAEB and ΔAFC,

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC = 90^\circ$ [Given: $BE \perp AC$]

[Given: $CF \perp AB$]

$AB = AC$ [Given]

$\Rightarrow \Delta AEB \cong \Delta AFC$ [AAS]

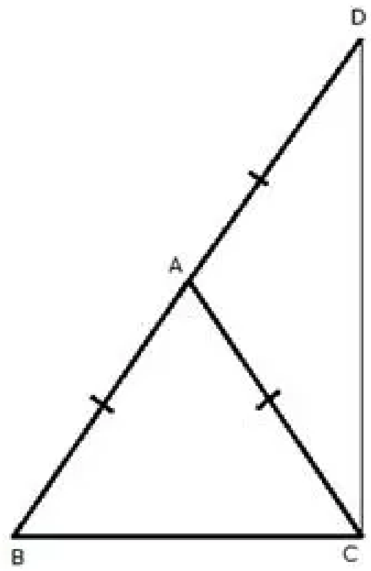
$\therefore BE = CF$ [cpct]

(ii)Since $\Delta AEB \cong \Delta AFC$

$\angle ABE = \angle AFC$

$\therefore AF = AE$ [congruent angles of congruent triangles]

Solution 4:



Const: Join CD.

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$.....(i) [angles opp. to equal sides are equal]

In ΔACD,

$AC = AD$ [Given]

$\therefore \angle ADC = \angle ACD$..........(ii)

Adding (i) and (ii)

$\angle B + \angle ADC = \angle C + \angle ACD$

$\angle B + \angle ADC = \angle BCD$.......(iii)

In ΔBCD,

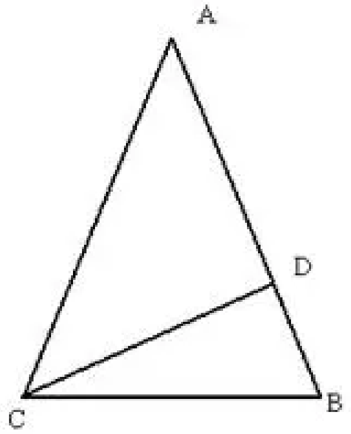
$\angle B + \angle ADC + \angle BCD = 180^\circ$

$\angle BCD + \angle BCD = 180^\circ$ [From (iii)]

$2\angle BCD = 180^\circ$

$\angle BCD = 90^\circ$

Solution 5:



AB = AC

ΔABC is an isosceles triangle.

$\angle A = 36°$

$\angle B = \angle C = \dfrac{180° - 36°}{2} = 72°$

$\angle ACD = \angle BCD = 36°$ [$\because$ CD is the angle bisector of $\angle C$]

ΔADC is an isosceles triangle since $\angle DAC = \angle DCA = 36°$

$\therefore$ AD = CD......(i)

In ΔDCB,

$\angle CDB = 180° - (\angle DCB + \angle DBC)$

$= 180° - (36° + 72°)$

$= 180° - 108°$

$= 72°$

ΔDCB is an isosceles triangle since $\angle CDB = \angle CBD = 72°$

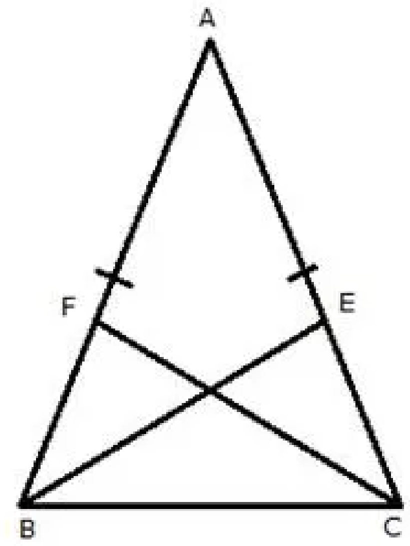
$\therefore$ DC = BC.......(ii)

From (i) and (ii), we get

AD = BC

Hence proved

Solution 6:



In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle B$

$\Rightarrow \angle BCF = \angle CBE$........(ii)

In ΔBCE and ΔCBF,

$\angle C = \angle B$ [From (i)]

$\angle BCF = \angle CBE$ [From (ii)]

$BC = BC$ [Common]

$\therefore \Delta BCE \cong \Delta CBF$ [AAS]

$\Rightarrow BE = CF$ [cpct]

Solution 7:

In ΔABC,

$AB = AC$ [Given]

$\therefore \angle ACB = \angle ABC$ [angles opp. to equal sides are equal]

$\Rightarrow \angle ABC = \angle ACB$......(i)

$\angle DBC = \angle ECB = 90^\circ$[Given]

$\Rightarrow \angle DBC = \angle ECB$ ....(ii)

Subtracting (i) from (ii)

$\angle DCB - \angle ABC = \angle ECB - \angle ACB$

$\Rightarrow \angle DBA = \angle ECA$.........(iii)

In ΔDBA and ΔECA,

$\angle DBA = \angle ECA$ [From (iii)]

$\angle DAB = \angle EAC$ [Vertically opposite angles]

$AB = AC$ [Given]

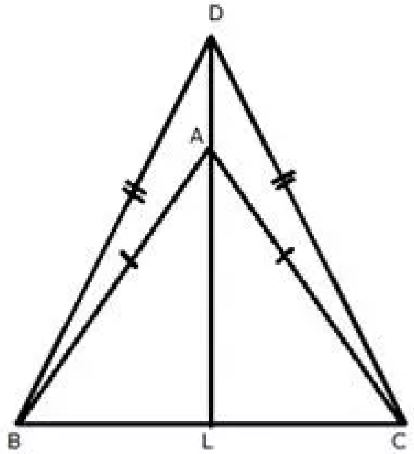
$\therefore \Delta DBA \cong \Delta ECA$ [ASA]

$\Rightarrow BD = CE$ [cpct]

Also,

$AD = AE$ [cpct]

Solution 8:



DA is produced to meet BC in L.

In ΔABC,

AB = AC [Given]

$\therefore \angle ACB = \angle ABC$.......(i) [angles opposite to equal sides are equal]

In ΔDBC,

DB = DC [Given]

$\therefore \angle DCB = \angle DBC$......(ii) [angles opposite to equal sides are equal]

Subtracting (i) from (ii)

$\angle DCB - \angle ACB = \angle DBC - \angle ABC$

$\Rightarrow \angle DCA = \angle DBA$.......(iii)

In ΔDBA and ΔDCA,

DB = DC [Given]

$\angle DBA = \angle DCA$ [From (iii)]

AB = AC [Given]

$\therefore \Delta DBA \cong \Delta DCA$ [SAS]

$\Rightarrow \angle BDA = \angle CDA$.........(iv) [cpct]

In ΔDBA,

$\angle BAL = \angle DBA + \angle BDA$.........(v)

[Ext. angle = sum of opp. int. angles]

From (iii), (iv) and (v)

$\angle BAL = \angle DCA + \angle CDA$.......(vi)

In ΔDCA,

$\angle CAL = \angle DCA + \angle CDA$..........(vii)

[Ext. angle = sum of opp. int. angles]

From (vi) and (vii)

$\angle BAL = \angle CAL$.......(viii)

In ΔBAL and ΔCAL,

$\angle BAL = \angle CAL$ [From (viii)]

$\angle ABL = \angle ACL$ [From (i)]

$AB = AC$ [Given]

$\therefore \Delta BAL \cong \Delta CAL$ [ASA]

$\Rightarrow \angle ALB = \angle ALC$ [cpct]

and $BL = LC$..........(ix) [cpct]

Now,

$\angle ALB + \angle ALC = 180°$

$\Rightarrow \angle ALB + \angle ALB = 180°$

$\Rightarrow 2\angle ALB = 180°$

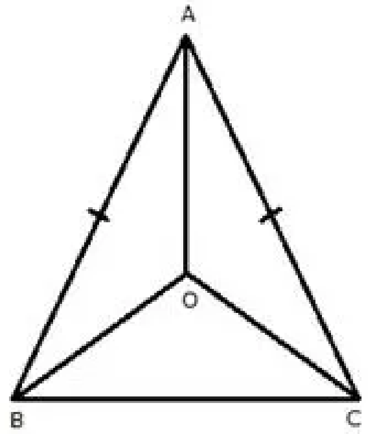
$\Rightarrow \angle ALB = 90°$

$\therefore AL \perp BC$

or $DL \perp BC$ and $BL = LC$

$\therefore$ DA produced bisects BC at right angle.

Solution 9:



In $\triangle$ ABC, we have AB = AC

$\Rightarrow \angle B = \angle C$ [angles opposite to equal sides are equal]

$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle C$

$\Rightarrow \angle OBC = \angle OCB$............(i)

$\Rightarrow$ OB = OC..............(ii)

[angles opposite to equal sides are equal]

Now,

In $\triangle$ ABO and $\triangle$ ACO,

AB = AC [Given]

$\angle OBC = \angle OCB$ [From (i)]

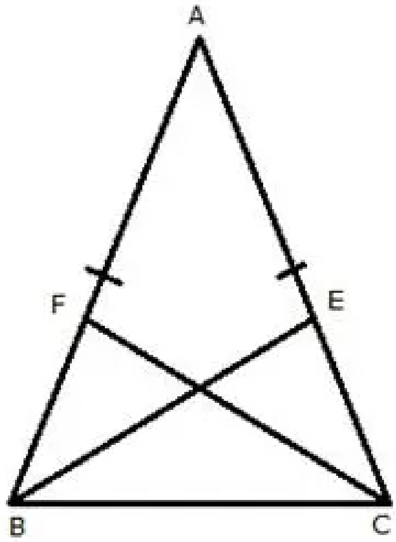
OB = OC [From (ii)]

$\Delta ABO \cong \Delta ACO$ [SAS criterion]

$\Rightarrow \angle BAO = \angle CAO$ [cpct]

Therefore, AO bisects $\angle$ BAC.

Solution 10:



In ΔABC,

$AB = AC$ [Given]

$\therefore \angle C = \angle B$......(i) [angles opp. to equal sides are equal]

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BF = CE$........(ii)

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BF = CE$........(ii)

In ΔBCE and ΔCBF,

$\angle C = \angle B$ [From (i)]

$BF = CE$ [From (ii)]

$BC = BC$ [Common]

$\therefore \Delta BCE \cong \Delta CBF$ [SAS]

$\Rightarrow BE = CF$ [cpct]

Solution 11:

In ΔAPQ,

$AP = AQ$ [Given]

$\therefore \angle APQ = \angle AQP$.......(i)

[angles opposite to equal sides are equal]

In ΔABP,

$\angle APQ = \angle BAP + \angle ABP$.......(ii)

[Ext. angle is equal to sum of opp. int. angles]

In ΔAQC,

$\angle AQP = \angle CAQ + \angle ACQ$.......(iii)

[Ext. angle is equal to sum of opp. int. angles]

From (i), (ii) and (iii)

$\angle BAP + \angle ABP = \angle CAQ + \angle ACQ$

But, $\angle BAP = \angle CAQ$ [Given]

$\Rightarrow \angle CAQ + \angle ABP = \angle CAQ + \angle ACQ$

$\Rightarrow \angle ABP = \angle CAQ + \angle ACQ - \angle CAQ$

$\Rightarrow \angle ABP = \angle ACQ$

$\Rightarrow \angle B = \angle C$.......(iv)

In ΔABC,

$\angle B = \angle C$

$\Rightarrow AB = AC$ [Sides opposite to equal angles are equal]

Solution 12:

Since AE || BC and DAB is the transversal

$\therefore \angle DAE = \angle ABC = \angle B$ [Corresponding angles]

Since AE || BC and AC is the transversal

$\angle CAE = \angle ACB = \angle C$ [Alternate Angles]

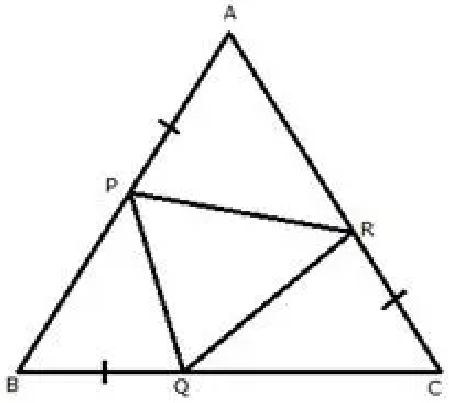
But AE bisects $\angle CAD$

$\therefore \angle DAE = \angle CAE$

$\Rightarrow \angle B = \angle C$

$\Rightarrow AB = AC$ [Sides opposite to equal angles are equal]

Solution 13:



AB = BC = CA.......(i) [Given]

AP = BQ = CR.......(ii) [Given]

Subtracting (ii) from (i)

AB - AP = BC - BQ = CA - CR

BP = CQ = AR ....(iii)

$\therefore$ $\angle A = \angle B = \angle C$ ....(iv) [angles opp. to equal sides are equal]

In ΔBPQ and ΔCQR,

BP = CQ [From (iii)]

$\angle B = \angle C$ [From (iv)]

BQ = CR [Given]

$\therefore$ $\Delta BPQ \cong \Delta CQR$ [SAS criterion]

$\Rightarrow$ PQ = QR........(v)

In ΔCQR and ΔAPR,

CQ = AR [From (iii)]

$\angle C = \angle A$ [From (iv)]

CR = AP [Given]

$\therefore$ $\Delta CQR \cong \Delta APR$ [SAS criterion]

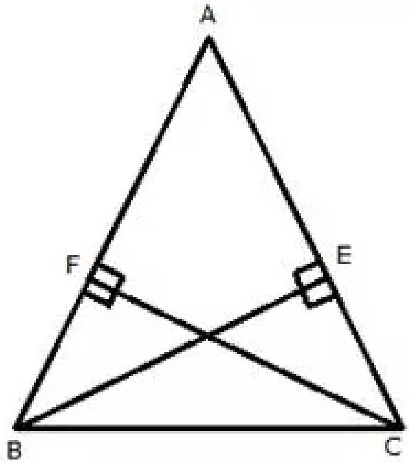
$\Rightarrow$ QR = PR........(vi)

From (v) and (vi)

PQ = QR = PR

Therefore, PQR is an equilateral triangle.

Solution 14:



In $\triangle$ABE and $\triangle$ACF,

$\angle A = \angle A$[Common]

$\angle AEB = \angle AFC = 90^0$[Given: BE $\perp$ AC; CF $\perp$ AB]

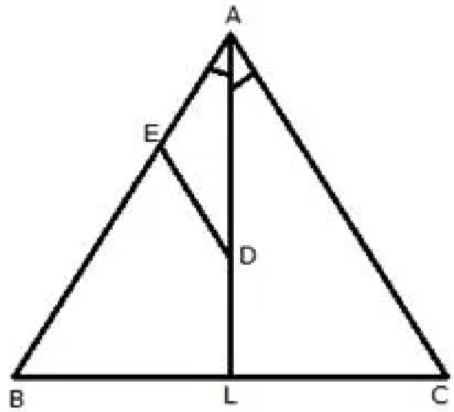
BE = CF[Given]

$\therefore \Delta ABE \cong \Delta ACF$ [AAS criterion]

$\Rightarrow AB = AC$

Therefore, ABC is an isosceles triangle.

Solution 15:



AL is bisector of angle A. Let D is any point on AL. From D, a straight line DE is drawn parallel to AC.

DE || AC [Given]

$\therefore$ $\angle$ADE = $\angle$DAC....(i) [Alternate angles]

$\angle$DAC = $\angle$DAE.......(ii) [AL is bisector of $\angle$A]

From (i) and (ii)

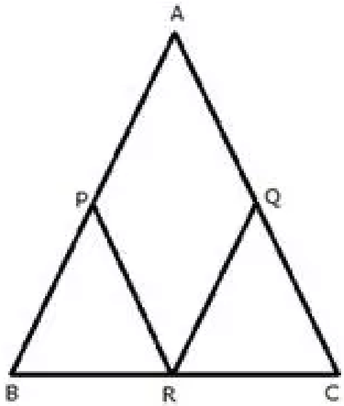
$\angle$ADE = $\angle$DAE

$\therefore$ AE = ED [Sides opposite to equal angles are equal]

Therefore, AED is an isosceles triangle.

Solution 16:

(i)



In Δ ABC,

AB = AC

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow$AP = AQ ....(i)[Since P and Q are mid - points]

In Δ BCA,

PR = $\frac{1}{2}$AC [PR is line joining the mid - points of AB and BC]

$\Rightarrow$PR = AQ........(ii)

In Δ CAB,

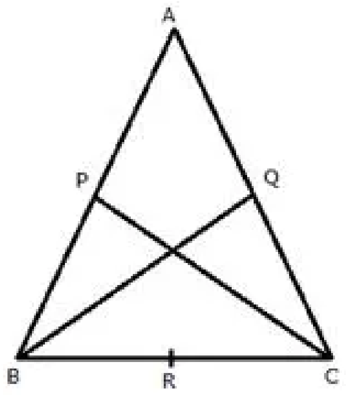
QR = $\frac{1}{2}$AB [QR is line joining the mid - points of AC and BC]

$\Rightarrow$QR = AP......(iii)

From (i), (ii) and (iii)

PR = QR

(ii)



AB = AC

$\Rightarrow \angle B = \angle C$

Also,

$\frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow BP = CQ$ [P and Q are mid-points of AB and AC]

In $\triangle BPC$ and $\triangle CQB$,

BP = CQ

$\angle B = \angle C$

BC = BC

Therefore, $\Delta BPC \cong \Delta CQB$ [SAS]

BP = CP

Solution 17:

(i) In Δ ACB,

AC = AC[Given]

$\therefore \angle ABC = \angle ACB$ ....(i)[angles opposite to equal sides are equal]

$\angle ACD + \angle ACB = 180^0$ ....(ii)[DCB is a straight line]

$\angle ABC + \angle CBE = 180^0$ ....(iii)[ABE is a straight line]

Equating (ii) and (iii)

$\angle ACD + \angle ACB = \angle ABC + \angle CBE$

$\Rightarrow \angle ACD + \angle ACB = \angle ACB + \angle CBE$[From (i)]

$\Rightarrow \angle ACD = \angle CBE$

(ii)

In ΔACD and ΔCBE,

DC = CB [Given]

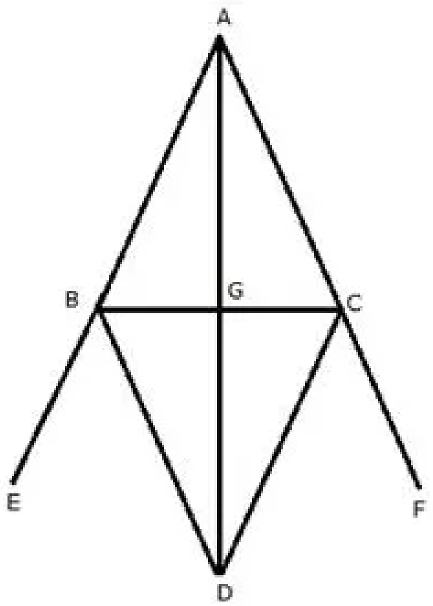
AC = BE [Given]

$\angle ACD = \angle CBE$ [Proved Earlier]

$\therefore \Delta ACD \cong \Delta CBE$ [SAS criterion]

$\Rightarrow$ AD = CE [cpct]

Solution 18:



AB is produced to E and AC is produced to F. BD is bisector of angle CBE and CD is bisector of angle BCF. BD and CD meet at D.

In $\triangle$ABC,

AB = AC[Given]

$\therefore \angle C = \angle B$[angles opposite to equal sides are equal]

$\angle CBE = 180^0 - \angle B$[ABE is a straight line]

$\Rightarrow \angle CBD = \frac{180^\circ - \angle B}{2}$ [BD is bisector of $\angle$ CBE]

$\Rightarrow \angle CBD = 90^\circ - \frac{\angle B}{2}$ ....(i)

Similarly,

$\angle BCF = 180^0 - \angle C$[ACF is a straight line]

$\Rightarrow \angle BCD = \frac{180^\circ - \angle C}{2}$ [CD is bisector of $\angle$ BCF]

$\Rightarrow \angle BCD = 90^\circ - \frac{\angle C}{2}$ ....(ii)

Now,

$\Rightarrow \angle CBD = 90^\circ - \frac{\angle C}{2}$ $[\because \angle B = \angle C]$

$\Rightarrow \angle CBD = \angle BCD$

In $\triangle$BCD,

$\angle CBD = \angle BCD$

$\therefore$ BD = CD

In $\triangle$ABD and $\triangle$ACD,

AB = AC[Given]

AD = AD[Common]

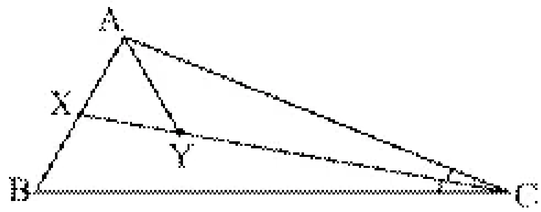
BD = CD[Proved]

$\therefore \Delta ABD \cong \Delta ACD$ [SSS criterion]

$\Rightarrow \angle BAD = \angle CAD$ [cpct]

Therefore, AD bisects $\angle$ A.

Solution 19:



In $\triangle ABC$,

CX is the angle bisector of $\angle C$

$\Rightarrow \angle ACY = \angle BCX$ .... (i)

In $\triangle AXY$,

AX = AY [Given]

$\angle AXY = \angle AYX$ ....(ii) [angles opposite to equal sides are equal]

Now $\angle XYC = \angle AXB = 180°$ [straight line]

$\Rightarrow \angle AYX + \angle AYC = \angle AXY + \angle BXY$

$\Rightarrow \angle AYC = \angle BXY$ .... (iii) [From (ii)]

In $\triangle AYC$ and $\triangle BXC$

$\angle AYC + \angle ACY + \angle CAY = \angle BXC + \angle BCX + \angle XBC = 180°$

$\Rightarrow \angle CAY = \angle XBC$ [From (i) and (iii)]

$\Rightarrow \angle CAY = \angle ABC$

Solution 20:

Since IA || CP and CA is a transversal

$\therefore$ $\angle$CAI = $\angle$PCA [Alternate angles]

Also, IA || CP and AP is a transversal

$\therefore$ $\angle$IAB = $\angle$APC [Corresponding angles]

But $\therefore$ $\angle$CAI = $\angle$IAB [Given]

$\therefore$ $\angle$PCA = $\angle$APC

$\Rightarrow$AC = AP

Similarly,

BC = BQ

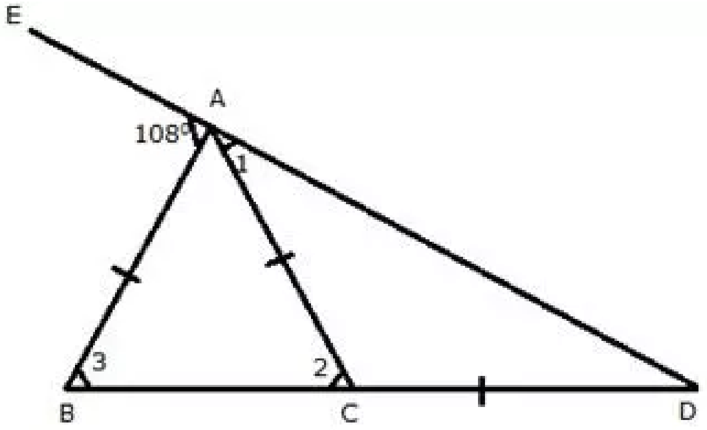
Now,

PQ = AP + AB + BQ

= AC + AB + BC

= Perimeter of $\triangle$ABC

Solution 21:



In $\triangle$ ABD,

$\angle BAE = \angle 3 + \angle ADB$

$\Rightarrow 108^0 = \angle 3 + \angle ADB$

But AB = AC

$\Rightarrow \angle 3 = \angle 2$

$\Rightarrow 108^0 = \angle 2 + \angle ADB$ ......(i)

Now,

In $\triangle$ ACD,

$\angle 2 = \angle 1 + \angle ADB$

But AC = CD

$\Rightarrow \angle 1 = \angle ADB$

$\Rightarrow \angle 2 = \angle ADB + \angle ADB$

$\Rightarrow \angle 2 = 2\angle ADB$

Putting this value in (i)

$\Rightarrow 108^0 = 2\angle ADB + \angle ADB$

$\Rightarrow 3\angle ADB = 108^0$

$\Rightarrow \angle ADB = 36^0$

Solution 22:

ABC is an equilateral triangle.

Therefore, AB = BC = AC = 15 cm

$\angle A = \angle B = \angle C = 60^\circ$

In ΔADE, DE || BC[Given]

$\angle AED = 60^\circ$ [$\because \angle ACB = 60^\circ$]

$\angle ADE = 60^\circ$ [$\because \angle ABC = 60^\circ$]

$\angle DAE = 180^\circ - (60^\circ + 60^\circ) = 60^\circ$

Similarly, ΔBDF & ΔGEC are equilateral triangles.

$= 60^\circ$ [$\because \angle C = 60^\circ$]

Let AD = x, AE = x, DE = x [$\because \Delta ADE$ is an equilateral triangle]

Let BD = y, FD = y, FB = y [$\because \Delta BDF$ is an equilateral triangle]

Let EC = z, GC = z, GE = z [$\because \Delta GEC$ is an equilateral triangle]

Now, AD + DB = 15 $\Rightarrow x + y = 15$......(i)

AE + EC = 15 $\Rightarrow x + z = 15$.....(ii)

Given, DE + DF + EG = 20

$\Rightarrow x + y + z = 20$

$\Rightarrow 15 + z = 20$ [from (i)]

$\Rightarrow z = 5$

From (ii), we get $x = 10$

$\therefore y = 5$

Also, BC = 15

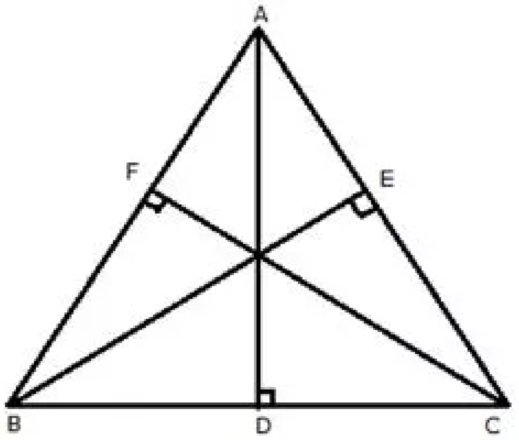
BF + FG + GC = 15

$\Rightarrow y$ + FG + z = 15

$\Rightarrow$ 5 + FG + 5 = 15

$\Rightarrow$ FG = 5

Solution 23:



In right $\triangle$ BEC and $\triangle$ BFC,

BE = CF[Given]

BC = BC[Common]

$\angle$BEC = $\angle$BFC[each = 90^0]

$\therefore \triangle BEC \cong \triangle BFC$ [RHS]

$\Rightarrow \angle B = \angle C$

Similarly,

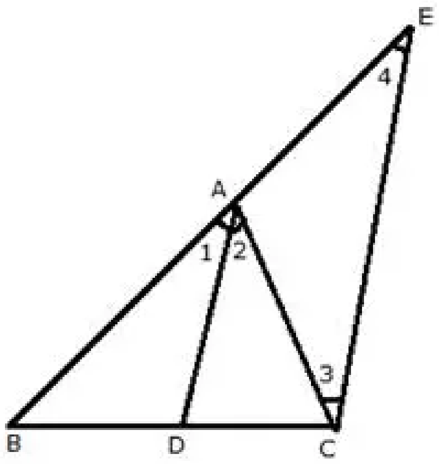
$\angle A = \angle B$

Hence, $\angle A = \angle B = \angle C$

$\Rightarrow AB = BC = AC$

Therefore, ABC is an equilateral triangle.

Solution 24:



DA || CE[Given]

$\Rightarrow \angle 1 = \angle 4$........(i) [Corresponding angles]

$\angle 2 = \angle 3$........(ii) [Alternate angles]

But $\angle 1 = \angle 2$.........(iii) [AD is the bisector of $\angle$ A]

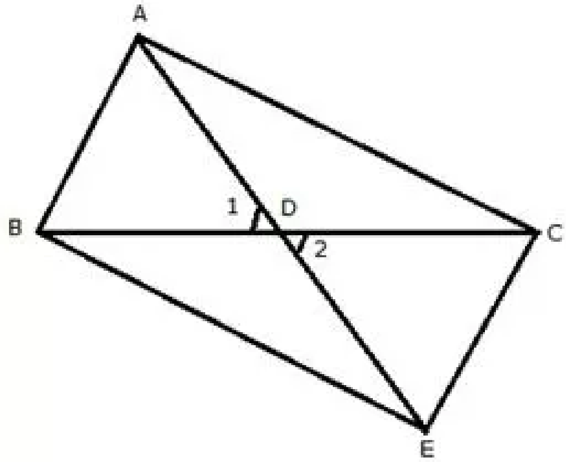
From (i), (ii) and (iii)

$\angle 3 = \angle 4$

$\Rightarrow$ AC = AE

$\Rightarrow \Delta$ ACE is an isosceles triangle.

Solution 25:



Produce AD upto E such that AD = DE.

In ΔABD and ΔEDC,

AD = DE [by construction]

BD = CD [Given]

$\angle 1 = \angle 2$ [vertically opposite angles]

$\therefore \Delta ABD \cong \Delta EDC$ [SAS]

$\Rightarrow$ AB = CE.......(i)

and $\angle BAD = \angle CED$

But, $\angle BAD = \angle CAD$ [AD is bisector of $\angle BAC$]

$\therefore \angle CED = \angle CAD$

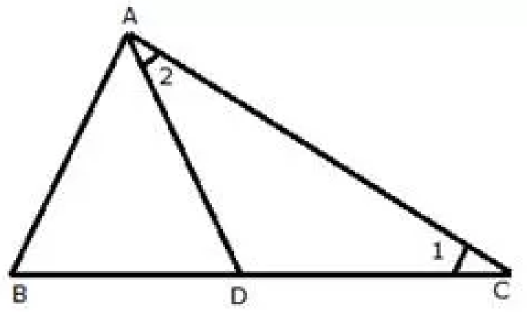
$\Rightarrow$ AC = CE......(ii)

From (i) and (ii)

AB = AC

Hence, ABC is an isosceles triangle.

Solution 26:



Since AB = AD = BD

$\therefore$ ΔABD is an equilateral triangle.

$$\therefore \angle ADB = 60^\circ$$
$$\Rightarrow \angle ADC = 180^\circ - \angle ADB$$
$$= 180^\circ - 60^\circ$$
$$= 120^\circ$$

Again in ΔADC,

AD = DC

$$\therefore \angle 1 = \angle 2$$

But,

$$\angle 1 + \angle 2 + \angle ADC = 180^\circ$$
$$\Rightarrow 2\angle 1 + 120^\circ = 180^\circ$$
$$\Rightarrow 2\angle 1 = 60^\circ$$
$$\Rightarrow \angle 1 = 30^\circ$$
$$\Rightarrow \angle C = 30^\circ$$
$$\therefore \angle ADC : \angle C = 120^\circ : 30^\circ$$
$$\Rightarrow \angle ADC : \angle C = 4 : 1$$

Solution 27:

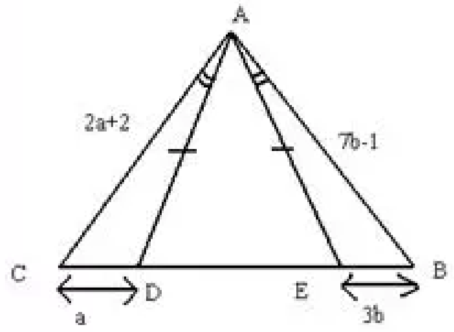
(i)

$$\text{In } \Delta CAE, \angle CAE = \angle AEC = \frac{180^\circ - 68^\circ}{2} = 56^\circ \ [\because CE = AC]$$

$$\text{In } \angle BEA, \ a = 180^\circ - 56^\circ = 124^\circ$$

$$\text{In } \Delta ABE, \angle ABE = 180^\circ - (a + \angle BAE)$$
$$= 180^\circ - (124^\circ + 14^\circ)$$
$$= 180^\circ - 138^\circ = 42^\circ$$

(ii)



In ΔAEB & ΔCAD,

$\angle EAB = \angle CAD$ [Given]

$\angle ADC = \angle AEB$ [$\because \angle ADE = \angle AED \{AE = AD\}$

$180^\circ - \angle ADE = 180^\circ - \angle AED$

$\angle ADC = \angle AEB$]

AE=AD[Given]

$\therefore \Delta AEB \cong \Delta CAD$ [ASA]

AC=AB[By C.P.C.T.]

$2a+2=7b-1$

$\Rightarrow 2a-7b=-3$.....(i)

CD=EB

$\Rightarrow a=3b$.......(ii)

Solving (i) & (ii), we get

$a=9$, $b=3$